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<u>FIRST PROMPT</u>

In a high school statistics course, students are studying inferential statistics. In particular, they are estimating the confidence interval for the proportion of voters in the population who are in favor of a particular candidate based on a sample of voters. The teacher has finished explaining the "conservative approach / worst-case scenario" for estimating the population standard deviation. Students are struggling to make sense of this idea. A student shares her confusion with the teacher:

"Okay, I get that maybe it's not a good idea not to use p-hat [the sample proportion] as an estimate for p [the true proportion] in the first formula, $\square=\square(1-\square)\square$. So you said to use the equation $\square=.5\square$ instead. It seems strange the new equation doesn't even have a "p" in it. And where does it come from? Is there a connection between the second equation and the first one, or are they two totally different things?"

How might the teacher respond to this student's questions?

<u>Possible Mathematical Foci</u>

- *The conservative estimate for the standard deviation of the true proportion assumes the situation of maximum variability for the parameter of interest, the true proportion.*
- *The conservative estimate for the standard deviation of the true proportion requires knowing only the sample size.*
- *The formula for the standard deviation of the true proportion parameter, $\square=\square(1-\square)\square$, is maximized when the true proportion is assumed to be equal to .5.*
    - This can be proven in a numerous ways, using reasoning based on arithmetic, geometry, and/or calculus.

SECOND PROMPT

In a lesson on linear systems of equations, the teacher is leading students through an activity in which they are summarizing their findings for linear systems with two equations and two unknowns. "We've seen three different cases. Let's try to summarize them in the table. … Okay, this problem is like the others we saw in which the solution is a line. When we solve these types of systems algebraically, we get a true equation—something like $-10=10$ or $0=0$. And what about the graph? We get one line for the graph. And how many points are on this line? Multiple points, right? I have one here and one here and one here [teacher is adding points to the line on the board]. In fact, there are many, many points on this line, right? So what I wind up saying about the solution to systems of equations that fall into this category is that the answer is 'all real numbers.'"

[Notes: The teacher's lack of precision in discussing this case appears to have led to some confusion on the part of her students. In particular, it is not clear that students see that it is the *intersection* of two *coincident* lines that has resulted in the solution being a line. (The summary table shows only one line; the teacher's explanation did not include discussion of *coincident* lines; and while the two equations making up the system were graphed on an overhead graphing calculator, the equations were graphed simultaneously, so it was not apparent that the graph consisted of coincident lines). Thus, it appears that some students see the "single line" solution as directly following from this case without connection to the idea of coincident lines. The teacher's statement that the solution is represented by "all real numbers" is incorrect. (In her mind, "infinitely many solutions" and "all real numbers" appear to be synonymous.) It is difficult to ascertain whether this error has created disequilibrium for students at this point in time; it is not hard to imagine, though, that this error may present difficulties for students in the future.]

Possible Mathematical Foci

- *A system of two linear equations in two variables will have infinitely many solutions if the equations are consistent and dependent.*
- *"Infinitely many solutions" is not necessarily synonymous with "all real solutions."*